## Death in Paris: How Mathematics Became an Art

Amir Alexander

re:place conference, Berlin, November 16, 2007

The young revolutionary was going through the papers of his martyred friend when he came across the following short verse:

L'éternel cyprès m'environne; Plus pâle que la pâle automne, Je m'incline vers le tombeau.

(The eternal cypresses surround me; Paler than the pallor of autumn, I bend down towards the grave.)

Haunting lines they were, and prophetic, for their author died shortly after penning them. In the course of a stormy political career on the radical fringes, the young poet was ensnared in a love triangle and challenged to a duel. On the dawn of May 30, 1832 he was shot in the stomach and left to die on an empty Paris street. His last words were directed at his younger brother, who was overcome with grief seeing him on his deathbed: "don't cry," he said to him; "I need all my courage to die at twenty." The tragic death of the brilliant young man was a stunning blow to his friends and fellow radicals. But in its way the young Frenchman's fate was emblematic of artistic souls in its time, the early 19<sup>th</sup> century. For in that age, often characterized as the apex of High Romanticism, a tragic life and/or early and sudden death were almost a biographical requirement for poets, artists, and musicians.

Lord Byron is perhaps the most famous, having died in 1824 at age 32 defending the liberty of the Greeks against the Turks. But he was hardly alone among the poets: his friend Percy Bysshe Shelley's death by drowning two years earlier was just as tragic, if less heroic, as was the untimely death of John Keats, who perished of tuberculosis in 1821 at the age of 25. The German poet and philosopher Novalis died at the tender age of 28, while his contemporary Holderlin lived a ripe old 73, but had gone insane, and lived the last 36 years of his life effectively a prisoner in a friend's house.

Musicians seemed equally victimized by the Fates. **Beethoven** suffered the ultimate torment of a composer by gradually losing his hearing, and **Chopin** died at 39, at the height of his success, of tuberculosis. **Mozart**, who died at age 35 in 1791, may have belonged to an earlier generation, but he was warmly embraced by the Romantics who saw him as one of their own. To them, his burial in an unmarked pauper's grave represented the ultimate tragic fate of an artist. In the age of High Romanticism, a tormented life followed by untimely death came to be seen as a good predictor for true genius trying to make its way in the world.

Nevertheless, the young man who felt surrounded by "eternal cypresses" and perished at 20 on a Paris street does stand out among the romantic heroes of his age in this crucial respect: he was neither a poet nor a musician, nor an artist of any sort, but a mathematician. His lasting fame rests not on romantic verses, nor on sublime symphonies. It is due rather to his contributions to an esoteric field of high mathematics, which even today can be appreciated only by a select highly trained few. His name was Evariste Galois, and he is widely considered one of the chief founders of modern mathematics. His work on the resolution of equations of the 5<sup>th</sup> degree (quintic equations) or higher led him to develop what has come to be known as "Galois Theory", the foundation of the modern theory of groups.

According to legend, Galois never received the encouragement and support that were his due from the French mathematical establishment. At age 16, Galois determined to leave the Lycee Louis le Grand early by enrolling in the Ecole Polytechnique, the most prestigious science and engineering school in Europe. Aware of his true capacities, Galois did not bother studying for the entrance exam which he considered petty and irrelevant, with the result that he was promptly turned down by the examiners. When he tried again a year later, in 1829, the result was the same; legend has it that Galois, frustrated by a question he considered particularly pointless lost his cool and hurled an eraser at the pedantic examiner.

Rejected by the school of his choice Galois begrudgingly enrolled in the Ecole Normale, a distant second at the time to the Ecole Polytechnique in prestige and rigor. He found

solace in radical politics, and when an insurrection broke out in July 1830 he saw it is his chance to strike a blow for the people against the oppressive Restoration regime. Here again, however, his desires were frustrated, for the Ecole's director, M. Guigniault, took the precautionary measure of locking his students behind the school's walls. Galois made every effort to scale the walls, all to no avail. And so, while the students of the Ecole Polytechnique led revolutionary bands to the barricades, Galois remained locked up, fuming and helpless, unable to take part.

Galois never forgave M. Guigniault for causing him to miss his chance at Revolution. The two soon locked horns once more when young Evariste published an article in the student newspaper denouncing the director as a coward and hypocrite. Guigniault demanded an apology. Galois refused, and was expelled from the school. In January of 1831, at the age of 18, he found himself alone and penniless on the streets of Paris.

Despite it all, and through all these turbulent years, Galois did not despair of interesting the leading mathematicians in his mathematical discoveries. Already in 1829 he sent a memoir to the illustrious Augustin Louis Cauchy, member of the Academy, professor at both the Ecole Polytechnique and the Faculty of Sciences at the University of Paris. Galois entertained high hopes that Cauchy, whose work he much admired, would recognize the brilliance of his own achievement. But his hopes were dashed once more, for Cauchy showed little interest in his manuscript, and then promptly lost it. Rather than hang his hopes again on the opinion of a single man, Galois then decided to submit his work to a competition organized by the Academy. The fates, however, seem to have

conspired against him. His entry was given to the distinguished but elderly Academician Joseph Fourier, who died before submitting his report on the work. When the memoir was not found among Fourier's belongings, Galois' name was eliminated from the competition.

Even this did not spell the end of the young mathematicians' hopes for recognition by his established elders. Fourier's colleague at the Academy, Denis Poisson, showed interest in Galois, and asked him to present him with a more comprehensive version of it. Galois' did so, and then waited anxiously for months to hear back from the Academician. The report, when it finally came, was another disappointment: Poisson could not make heads or tails of Galois' reasoning and asked for a fuller exposition. Galois was crushed.

Out on the streets of Paris, the radical firebrand Galois was faring no better. In 1831 he joined the Artillery of the National Guard, a radical militia that was banned by the new regime of Louis-Philippe. In April he took part in a raucous celebration at a tavern to mark the acquittal and release from prison of nineteen of its officers. According to the novelist Alexandre Dumas, who was present, Galois proposed a toast "to Louis-Philippe," but held up an open dagger along with his glass. Galois was arrested within days, and spent two months at Ste. Pelagie prison awaiting trial. When this came, he claimed that his actual toast was "to Louis Philippe, if he betrays!" Since he also claimed that he expected that the king would indeed betray, this proved to be less than a sterling defense. He was nevertheless acquitted by a sympathetic judge.

Free again, Galois managed to stay out of trouble for less than two months. On Bastille day, July 14, 1831, Galois was detained by the police once more while leading marchers to plant liberty trees on the Place de Grève. At the time of his arrest he was dressed in the uniform of the banned Artillery of the National Guard, and was carrying a loaded carbine, two pistols, and a knife. Sent once again to Ste. Pelagie, Early in 1832 the prison was struck with a cholera epidemic, and Galois was moved to a nearby convalescent home for the remainder of his sentence. While there, it appears that the young mathematician fell in love with Stephanie du Motel, the daughter of the institution's owner. While the details are sketchy, some hints in his letters imply that the ardent but inexperienced Galois gave offense to the young lady, who called on two of her male friends for redress. When attempts at mediation failed, Galois was challenged to a duel, which he duly accepted.

By this time Galois had been a free man once more for several weeks, but his prospects seemed as bleak as ever. Whether he survived the duel or not seemed hardly to matter to Galois, who felt surrounded by insurmountable obstacles. Hemmed in on all sides by "eternal cypresses," the twenty year old concluded that his only hope was to cast his lot with posterity. Only those unknown men of the future could grant him the recognition he so richly deserved, but that was denied him by his contemporaries.

And so, on the night of May 29, 1832, Galois set out to compose his testament to posterity. He wrote several farewell letters to his friends pleading his case and asking them not to blame his opponents in his death. He then composed a long memoir summarizing his mathematical work and sent to his friend and fellow radical Auguste

Chevalier – the man who some days later would discover the poem among his papers. Galois, as we know, was shot the next morning and died in the arms of his tearful brother. Chevalier was left with his testament of genius, and true to his promise, persisted in his efforts to interest the scientific establishment in Galois' work. He did not rest until, decades later, he was able to grant to his fallen friend the ultimate gift that had eluded him in his lifetime: the recognition and admiration of the leading mathematicians of the day, and ultimately – immortality.

So goes the legend of Galois as it has come down to us. Its originator was undoubtedly Galois himself, who railed against his oppressors and saw himself as the tragic victim of institutional mediocrity and indifference. It was carried on and elaborated on by his radical friends, chief among them the brothers Auguste and Ernest Chevalier, and Galois' own brother Alfred. When Galois' works were ultimately published in 1846 by the Academician Joseph Liouville, and his mathematical accomplishments broadly acknowledged, the story was carried on to the halls of the French scientific establishment. In 1896, to mark the centennial of the Ecole Normale, the Ecole's inspector general, Paul Dupuy wrote a comprehensive and detailed biography of the young man who was expelled from the school 65 years before. When in 1909 a commemorative plaque was placed on the house of Galois' birth in the Paris suburb of Bourg la Reine, a representative of the Ecole Normale formally apologized "to the genius of Galois, in the name of that school which he entered with regret, where he was misunderstood, from which he was driven out, and to which he is, despite everything, one of the most brilliant

glories." The very institutions that had spurned him in his lifetime now adopted the story of Galois' life as the purest embodiment of genius.

The embrace of Galois by the French scientific establishment posthumously fulfilled his life's dream, but it did not mark the limit of his spreading fame. In the 1920's the remarkable life story of Galois was introduced to the English speaking world by George Sarton, Belgian ex-patriate and founder of the modern discipline of the history of science. It was then taken on by Eric Temple Bell – science fiction author, aspiring poet, and Caltech mathematician, who immortalized Galois in his 1937 biographical collection *Men of Mathematics*. In a chapter entitled "Genius and Stupidity" Bell presented Galois's life as an epic struggle of pure genius against stubborn mediocrity. Young Evariste, according to Bell, "beat his life out fighting one unconquerable fool after another."

Bell's book became an immediate best-seller, and the section devoted to Galois' tragic life was its most popular chapter by far. It became almost commonplace for mathematicians and physicists who came of age in the following decades to declare that they were first drawn to mathematical studies by reading Bell's account of Galois' life. It remained to this day as shared lore in academic mathematics departments, a story oft told to students as the embodiment of what it means to be a true mathematician.

The legend of Galois was and remains dramatic, captivating, and enormously appealing, as evidenced by the enduring popularity of Bell's telling of the story. It does however suffer from a serious, even debilitating flaw: It is not true. A close examination of the

available evidence shows that some of the most dramatic episodes in his legends are highly exaggerated if not downright false. The notion, for example, that Galois' mathematical legacy was contained in the letter he wrote on his last night is clearly misplaced. In fact the letter included a condensed version of the memoir that had so puzzled Poisson the previous year. This memoir, along with six articles Galois had published in his last two years, are the true basis of what became known subsequently as "Galois Theory." The notion that Cauchy, the leading mathematician of the day, turned a cold shoulder to Galois and lost his memoir is also highly questionable. There is good evidence that Cauchy encouraged his young compatriot and was prepared to present his work to the Academy, only to delay his report when Galois promised to produce a more complete version of his work. In fact, despite Galois' deep sense of persecution, he was clearly regarded as a promising mathematician by the leading mathematicians in Paris, was well known to many of them, and had published his works in prestigious journals alongside theirs. And how many aspiring mathematicians can claim to have accomplished as much by the age of 20?

There is little doubt, in fact, that the chief culprit in Galois' difficulties was Galois himself. For Evariste was a slight young man with a remarkable talent to provoke potential allies and turn friends into enemies. For a sense of how the young mathematician appeared to disinterested observers, consider this passage in a letter from Sophie Germain (the only woman in the club of elite mathematicians) to the up and coming Italian mathematician, Guillaume Libri: Your preoccupation, that of Cauchy, the death of M. Fourier, have been the final blow for this student, Galois who, in spite of his impertinence, showed signs of a clever disposition. All this has done so much that he's been expelled from l'École Normale. He is without money, and his mother has very little also. Having returned home, he continued his habit of insult, a sample of which he gave you after your best lecture at the Academy. The poor woman fled her house, leaving just enough for her son to live on, and has been forced to place herself as a companion in order to make ends meet. They say he will go completely mad. I fear this is true.

But the legend of Galois as endorsed by the Ecole Normale and disseminated to the world by Bell retained nothing of his obnoxiousness, his "habit of insult," or his cruelty to his mother. All that remained was the story of pure genius, persecuted by a fossilized establishment and driven to an early death – the perfect romantic legend.

Strange though it seems, Galois was not the only young mathematician destined to tragic immortality in early 19<sup>th</sup> century Paris. In 1826, as Galois was embarking on his career of "impertinence" at the Lycee Louis le Grand, a 24 year old Norwegian was also trying to make a name for himself in the scientific capital of Europe. His name was Nils Henrik Abel, and he too, like Galois, was having a difficult time. Abel came from a respectable middle class family, and was introduced to mathematics during his studies at the Cathedral School in Christiania (modern Oslo). His teacher, Bernt Michael Holmboe,

recognized Abel's remarkable talents and introduced him to the exclusive circle of Norway's scientific elite.

Abel's fortunes took a serious turn when his father's political career ended in disgrace and he died soon afterwards, leaving Nils Henrik, his mother, and his sisters without a source of income. From that time onwards Abel suffered the pressures of being sole breadwinner for his family, while at the same pursuing a financially unrewarding mathematical career. After graduating from the Cathedral School Abel subsisted on meager fellowships awarded by the University of Christiania, and occasionally the personal generosity of Norway's leading scientist, Christoffer Hansteen, and his colleagues.

In 1825 Abel was one of a group of promising young Norwegian sent out by their government on a journey to European capitals, to establish connections and make a name for themselves. Abel's first stop was Berlin, where he befriended August Leopold Crelle, who became the most important scientific connection of his life. That very year Crelle founded the *Journal für reine und angewandte Mathematik*, which became popularly known as "Crelle's journal" and remained the foremost publication in pure mathematics throughout the 19<sup>th</sup> century. The very first issue contained no less than seven articles by Abel, and the following issues included even more. Meanwhile the professor of mathematics retired in Christiania, and Abel was a leading candidate to replace him. In the end, however, the position went to his friend and mentor Holmboe, who was older and also in Christiania at the time, whereas Abel was abroad.

In the summer of 1926 Abel finally arrived in Paris, his primary destination, but here his luck seems to have run out. Despite his keen efforts to make the acquaintance of the leading Parisian mathematicians, no one in the capital of scientific Europe seemed particularly interested in the work of the unknown Norwegian. In an eerie prelude to Galois' experience, a memoir that he gave Cauchy was lost, and not heard of again until after Abel's death. Disillusioned and in poor health, Abel returned to Berlin, where the attentions and companionship of Crelle restored his spirits. Nevertheless, despite Crelle's strong urging that he remain in Berlin, and even take over the editorship of the *Journal*, Abel headed back home.

When he arrived back in Christiania in the spring of 1827, Abel's international reputation was clearly on the rise. *Crelle's Journal* was quickly establishing itself as the leading mathematical publication in Europe, and Abel's numerous articles in it caught the attention of leading mathematicians. In particular, the great Adrien-Marie Legendre, who had ignored Abel in Paris, now began a prolonged correspondence with him which was to last for the rest of the young Norwegian's life. The following year Legendre joined three other French Academicians in a letter to the King of Sweden, urging him to provide for the brilliant mathematician who resided in his lands and appoint him to the Stockholm Academy. Crelle, meanwhile, never desisted from his efforts to bring Abel to Berlin, and was using all his connections in the Prussian government to establish a mathematical institute especially for him.

But despite the efforts of his influential friends, Abel's life changed little. He still subsisted on a combination of fellowships, substitute teaching at the university, and tutoring, and still supported his mother and sisters from his meager income. In addition, he was producing a steady stream of articles which enhanced his stature as a mathematician, but did little to improve his financial situation. "I am as poor as a church mouse" he once complained to his friend, Mrs. Hansteen.

Abel spent the Christmas of 1828 with the wealthy Smith family in the town of Froland, where his fiancée Crelly served as a governess. On January 9, when he was set to return to his teaching duties in Christiania he fell violently ill, coughing and spitting blood. For the next twelve weeks Abel lay sick in Froland, cared for by Crelly and the Smith family. Diagnosed with "galloping consumption," or pulmonary tuberculosis as it is known today, he died on April 6, 1829, at the age of 26. A letter from Crelle, announcing that his appointment in Berlin had finally been secured, arrived only a few days later. The following year the French Academy awarded its grand prize in mathematics jointly to Abel and Jacobi, with Abel's portion of the prize going to his widowed mother.

Abel had endured a life of economic hardship, and his death at the age of 26 was undoubtedly tragic. But unlike Galois, Abel never considered himself a victim of persecution. Nor was he: his scientific friends in Christiania went out of their way to secure his future, often delving into their own pockets to do so; four distinguished French Academicians had banded together to urge his appointment to the Swedish Academy, and Crelle in Berlin was working incessantly, and ultimately successfully, on his behalf. At

the time of his death several other scientific centers across Europe were showing interest in recruiting the young Norwegian, who was clearly a rising star in the mathematical firmament. There is no doubt that had Abel lived even a short while longer, he would have secured a position and an income worthy of his talents.

But all this did not prevent the uproar that immediately followed Abel's death: "Who killed Abel?" the cry resonated from Christiania, to Berlin, and to Paris. In obituaries, memoirs, and biographies, those who knew Able and – even more so – those who did not, competed with each other in portraying Abel as an innocent genius, "too good for this world," exploited and victimized by scheming mediocrities. In Christiania the chief culprits were thought to be the member of the science faculty, who never found a permanent position for Abel, and poor Holmboe, who took the professorship that was now seen as Abel's by right. In Paris the culprits were the Academicians, the ever present Cauchy in particular, who reputedly ignored the young genius and lost his work. Many years later the French scientist and statesman Vincent Raspail, who met Abel during his Paris sojourn, even claimed that Abel was so poor he had to walk from Paris to Christiania, while the rapacious Cauchy was fattening himself on multiple exorbitant incomes.

Abel himself would undoubtedly have been bewildered by these depictions, which posthumously transformed him into a tragic romantic hero. In life Abel was worldly enough to ingratiate himself at a young age with his country's intellectual elite, and amiable enough that during his tour of Europe he spent months traveling with his friends

in Italy before turning to Paris, his proper destination. In his legend, however, he became an alienated loner, ill at ease in the company of his peers and hopelessly impractical. His early death, which in truth was the tragic result of an incurable TB infection, was now seen to stem from his mistreatment by the European scientific establishment.

In life the amiable and conventional Abel and the fiery radical Galois could not have been more different. In death, however, they became virtual twins. Both struggled for recognition of their gifts in the city at the center of the world; both were turned away coldly by lesser men, setting them on a path to alienation and early death. In the eyes of posterity, both came to embody the tragedy of pure and innocent genius crushed by mediocrity and pettiness. Like poets and musicians of their time who endured similar travails and died young, the Frenchman and the Norwegian became tragic icons of the romantic imagination.

While Galois and Abel are likely the most famous tragic mathematicians whose popular biographies fit the mold of the romantic hero, they were far from the only ones: the Hungarian nobleman Janos Bolyai was born the same year as Abel, and is one of the chief founders of the radical new field of non-Euclidean geometry. He was 29 years old when, much like Galois and Abel, he too was crushed by the rebuff afforded him by a leading mathematician of the day, Carl Friedrich Gauss. After failing to interest that "prince of mathematicians" in his discoveries, he spent the rest of his life in retirement on his estate, without producing any more mathematical work.

The persistence of this "tragic" tradition is evident in the stories of later generations of mathematicians as well. Georg Cantor (1845-1918) is the founder of Set Theory, which is considered the logical foundation of mathematics as a whole. He was denied the recognition he deserved for his revolutionary work by the enmity of the powerful Leopold Kronecker (1823-1891), who was editor of *Crelle's Journal*. Cantor spent his days teaching at a provincial university, suffering repeated mental breakdowns, and ended his life in an insane asylum. Srinivasa Ramanujan (1887-1920), an Indian genius with an uncanny intuition for numbers, spent several years in Cambridge in the company of G. H. Hardy, but died penniless and alone in India in 1920. Kurt Gödel (1906-1978) turned both mathematics and philosophy on its head in 1930 with his "Incompleteness Theorem", but died alone of self-starvation in a New Jersey hospital nearly half a century later. The young Englishman Alan Turing (1912-1954), is considered the father of modern computer science and played a major role in the defeat of Nazi Germany. Persecuted for his homosexuality, he was hounded to his early death by an ungrateful nation. Alexander Grothendieck, born in 1928 and considered one of the greatest and most influential mathematicians of the 20<sup>th</sup> century, spent years in personal and political confrontations with fellow mathematicians. In 1991, at the age of 63, he left his home in Montpellier and disappeared. Most recently the portrayal of mathematician John Nash in the movie A *Beautiful Mind* presents the same popular image of the mathematician as a mentally fragile genius.

From the age of Galois and Abel to our own time, then, the ideal mathematical life has become indistinguishable from the ideal life of a poet, musician or artist. If Galois and

Abel have become the embodiment of the tragic mathematician, then Van Gogh probably represents the Platonic ideal of a tragic artist. After years of trying unsuccessfully to make his name in Paris and the provinces, he ultimately commits suicide, without having sold a single painting in his entire life. Much like Galois, only posterity fully appreciated the genius of Van Gogh, who was cruelly snubbed by his contemporaries.

It is important to note that the popular image of the mathematical tragic genius was an innovation of the early 19<sup>th</sup> century, and contrasted sharply with an older tradition of representation. The leading 18<sup>th</sup> century mathematicians who preceded the generation of Galois and Abel showed no tragic inclinations whatsoever. In fact, they tended to be powerful public figures, highly placed within the intellectual establishment of their time, and were often active members of the political class as well. The mathematical Bernoulli clan of Basel, for example, was so successful that it installed family members in the most prestigious mathematical chairs in Europe for three generations(!). Jean LeRond d'Alembert was a leading "philosophe," a stalwart of the fashionable salons, lifelong member of the Royal Academy of Sciences, and perhaps most significantly – co-editor of the Encyclopedie. Pierre-Louis Moreau de Maupertuis was the heroic leader of a geographical expedition and later president of the Berlin Academy and friend of Frederick II of Prussia. And Leonhard Euler, perhaps the greatest and certainly the most prolific of 18<sup>th</sup> Century mathematicians, held mathematics chairs in St. Petersburg, Berlin, and again St. Petersburg, and was a personal acquaintance and correspondent of Kings, Emperors, and Princes. One can hardly imagine a starker contrast than that

between the successful men of affairs of the 18<sup>th</sup> century, and the tragic alienated loners of the early 19<sup>th</sup> century.

At the same time, the new 19<sup>th</sup> century ideal of the mathematician as a tragic genius also marked a clear break between mathematics and contemporary natural sciences. The great "Men of science" of the 19<sup>th</sup> century were sometimes considered eccentric, but they had nothing of the tragic about them. In France leading scientists from Joseph Fourier (1768 – 1830) and Georges Cuvier (1769-1832) at the dawn of the century, to Henri Poincare (1754-1912) at its close, carried on a tradition of public service, becoming high officials in the service of the state. In Britain William Thomson (1824-1907), later Lord Kelvin, was not only a scientific innovator and university professor at age 22, but also an engineer who played a leading role in the laying of the first transatlantic cable and ultimately a gray eminence of British science. James Clerk Maxwell (1831-1879) also enjoyed success and acclaim early in life, and though he died at a relatively young age no tragic legend ever attached itself to his name. Charles Darwin (1809-1882), though something of a recluse at his country house for the later part of his life, had nevertheless been a member of the English scientific elite since his twenties, was happily married and had ten children.

In Germany Hermann von Helmholtz (1821-1894) rose quickly through the academic ranks and ultimately became a public spokesman for science of world renown. Albert Einstein, the greatest name in 20<sup>th</sup> century science, was often depicted as somewhat offbeat, but was certainly not tragic. Recognized for his achievements at a young age,

Einstein became a universal icon of scientific genius and was deeply involved in public and political causes. Evidently, the natural scientists of the 19<sup>th</sup> and 20th centuries, just like mathematicians of an earlier age, were viewed as successful men of affairs and leading members of the public sphere.

But in the early 19<sup>th</sup> century the popular image of the ideal mathematician took on a startlingly different character. In place of the engaged and publicly active savant, a tragic loner emerged, misunderstood and victimized by a heartless world. It was a shift embodied in the tragic legends that gained currency and even a canonical status among mathematicians and the broader public. Abel's fruitless struggles for recognition among the Parisian scientific elite, his poverty and early death, came to represent the hardships borne by pure mathematical genius. Galois' troubled and violent career was an even more dramatic testament to the tragic fate of genius shunned by an indifferent world. Other romantic heroes of mathematics, including Bolyai, Kantor, Ramanujan, and Turing, soon followed.

With the deaths of Galois and Abel, mathematics broke away from its traditions and charted a new course. No longer was mathematics viewed as an integral part of the natural sciences, its close companions for millennia. In the popular imagination, those who made mathematics their vocation now shared more with artists, poets, and musicians than with scientists. The legends of Galois, Abel, and Bolyai bear an unmistakable family resemblance to the widely known tales associated with Byron, Mozart and Van Gogh. But they are of a wholly different breed than the popular image of 19<sup>th</sup> century scientific

giants such as Thomson, Darwin, or Helmholtz. In spite of the essential role mathematics continued to play in the development of the natural sciences, the practitioners of mathematics came to be seen as fundamentally different than those engaged in all other scientific fields. In the early 19<sup>th</sup> century those who dedicated themselves to the study of higher mathematics came to be seen more as romantic artists than as objective and grounded scientists. And as mathematicians became artists, their craft became, by implication, an art.

But what does it mean for mathematics to be considered an "art" rather than a "science?" Perceptions of mathematicians and popular legends can undoubtedly shift, alienated loners supplant active men of affairs, but what does that imply for actual mathematical work? Does the technical practice of mathematics, the most timeless of disciplines, change along with popular mathematical imager? The answer is unequivocally yes. For the practice of "artistic" mathematics, as it emerged at the time of Galois and Abel, was profoundly different from the traditional practices of "scientific" mathematics.

The early 19<sup>th</sup> century is sometimes referred to as the period of the "re-birth" of mathematics. There is good reason for this: a new insistence on logical rigor and internal consistency pervaded the field, surpassing anything that had gone before. In the 18<sup>th</sup> century mathematicians were usually content to reach correct results, ignoring methodological difficulties along the way. If the end result proved true, the reasoning went, then the method had to be essentially correct as well. Such an approach is

characteristic of natural scientists to this day, who generally regard mathematics as a tool for achieving correct results about the world.

But to the new mathematicians of the 19<sup>th</sup> century this approach seemed dolefully inadequate. Mathematical reasoning, they argued, must be clear and transparent, or it was no mathematics at all. Mathematical techniques, such as the calculus, whose foundations were suspect, may be useful tools in the near term, but eventually they were bound to lead to error if the basis of their effectiveness was not clarified. Because of this, the main concern of 19<sup>th</sup> century mathematicians was not finding new results, but rather clarifying and systematizing the internal structure of mathematics itself. This has largely remained the concern of professional mathematicians to this day, and it is no wonder that the early 19<sup>th</sup> century is often viewed as the time of the birth of modern mathematics. But it also signaled a clear break between mathematics and the natural sciences. Whereas the sciences continued to view mathematics as a useful tool for describing and interrogating the physical world, the discipline of mathematics itself turned inwards and focused on investigating its own perfect edifice, unconstrained by the demands of physical reality. "Scientific" mathematics continued to flourish as part and parcel of the investigation of nature, but in the citadels of pure mathematics 19<sup>th</sup> century practitioners began reaching beyond the confines of the natural world, towards the transcendent world of mathematics itself.

In essence, whereas the great 18<sup>th</sup> century masters saw mathematics as inseparable from the physical world, 19<sup>th</sup> century mathematicians radically divorced mathematics from the

world. Mathematics now became its own separate universe – perfect, logical, consistent, and beautiful – and very different from the imperfect unpredictable universe we see around us. Whereas our own world is governed by the unyielding realities of physical nature and the contingencies of human existence, the mathematical world knows no such limitations. Its truths are transcendent, eternal, and perfect, existing on a different plane of reality than anything we see around us.

Is it a wonder then that the mathematics of the 19<sup>th</sup> century required a very different practitioner than the mathematics of earlier generations? As long as mathematics was part of the physical world, it was only natural to expect that a practicing mathematician would be part of this "real" world as well. 18<sup>th</sup> century mathematicians studied the physical world intensely, based their mathematical knowledge on their understanding of the world, and their physical theories on their understanding of mathematics. Both professionally and personally they were "men of the world" – those who possessed a far deeper understanding of the world we live in than any among us. Such men who immersed themselves in the study of reality could be expected to feel at home in the world. Their long and prosperous careers are sure testimony that d'Alembert, Euler, and their colleagues were indeed very comfortable in our mundane universe.

Things were very different, however, in the 19<sup>th</sup> century, when mathematics seemed to exist in a universe separate from our own, with its own rules and its own realities. Mathematicians now were not those with a special and deep understanding of our own world, but those unaccountably gifted with privileged access to an alternative and higher

reality. They were in a way, prophets, lonely emissaries from what Crelle, in his obituary to Abel called "the land of truth, mathematics." Only a select few could ever hope to gain a glimpse of this land of marvels and perfection, so different from our own.

In this regard, the tragic geniuses of 19<sup>th</sup> century mathematics are close kin to the tragic poets and artists of high romanticism. Byron and Shelley, as well as Beethoven and Chopin, delved into the sublime, a separate realm of pure beauty and perfection. Their inspired creations did not belong to our own paltry reality, but came to us from other lands, where truth and beauty reigned. These messages from the sublime could take the form of musical, poetic, or artistic creations, but beginning at this time they could take the form of mathematical creations as well. No less so than their artistic brethren, the mathematicians of this age were striving to catch a glimpse of a sublime land of perfection, and bring reports of its brilliance to those who had remained behind.

Is it a wonder that those blessed few who were able to glimpse the dazzling beauty of the mathematical universe would find life on Earth burdensome and confusing? Hardly. They are like the prisoner in Plato's cave metaphor, who climbs out from the shadows and sees the true Forms in all their brilliance. They will never again be at home in our flawed and compromised universe. Galois and Abel, like Byron or Beethoven, were creatures of a higher and better universe that most of us never glimpse. They did not belong in our physical world, with its contingent realities, politics, and power structures. The fact that they had to live their mortal lives in the mundane circumstances of 19<sup>th</sup> century Europe was, simply put, a tragedy. Inevitably, it ended with disillusionment and an early death.

In 1832 a 20 year old man was left to die on an empty Paris street, but his passing signified not only the end of a young life but a new beginning for mathematics. What had been a scientific study of the world, now became an artistic striving for sublime perfection. It is a tragic course, full of longing and despair, and Galois was one of its first victims. It is only fitting that his short life became the iconic tale of the new artistic mathematics, and he himself – its patron saint.